Row-major order and column-major order

widely-used strategies to store multidimensional arrays in linear storage

0	1	2
3	4	5

A_{00}	A_{01}	A_{02}
A_{10}	A_{11}	A_{12}

Note: zero-indexing

E. W. Dijkstra, "Why numbering should start at zero" (1982)

Row-major order

0	1	2	3	4	5

Column-major order

0	3	1	4	2	5

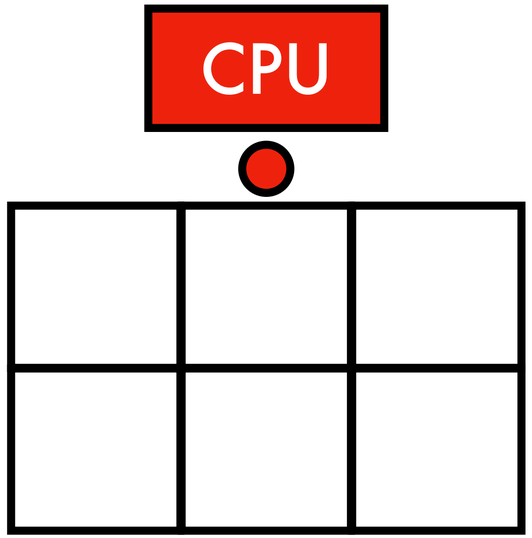
Lexicographic order

A_{00}	A_{01}	A_{02}	A_{10}	A_{11}	A_{12}

Colexicographic order

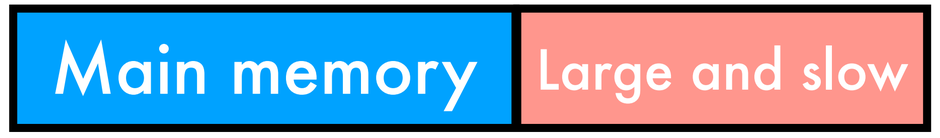
A_{00}	A_{10}	A_{01}	A_{11}	A_{02}	A_{12}

Why is storage order important? Caching



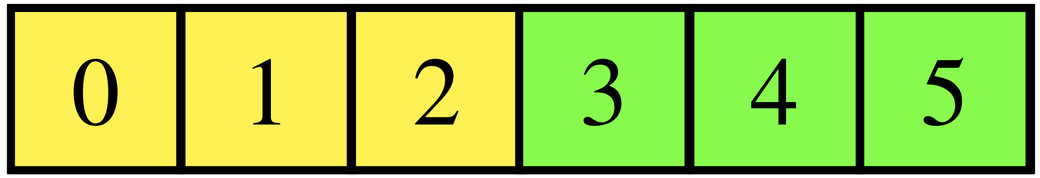
cache miss!

printout of values: 0 1 2 3 4 5

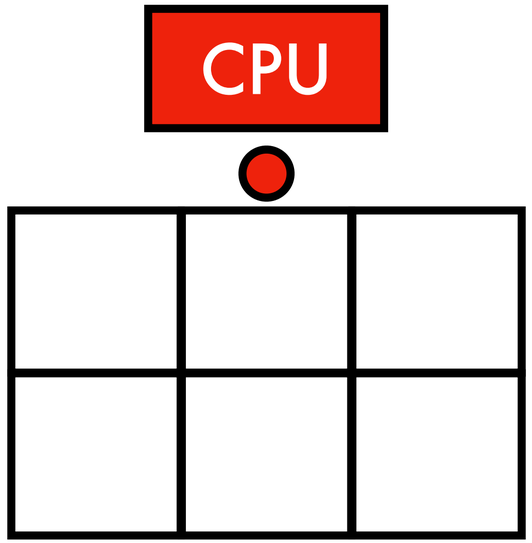


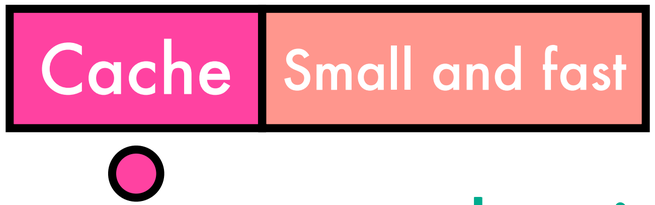
(spatial locality)

cache misses: 2



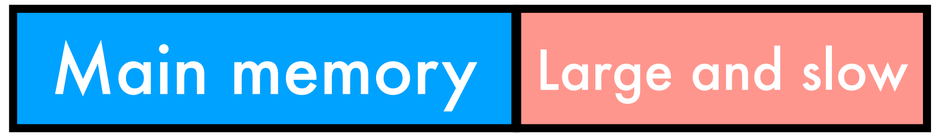
Row-major order



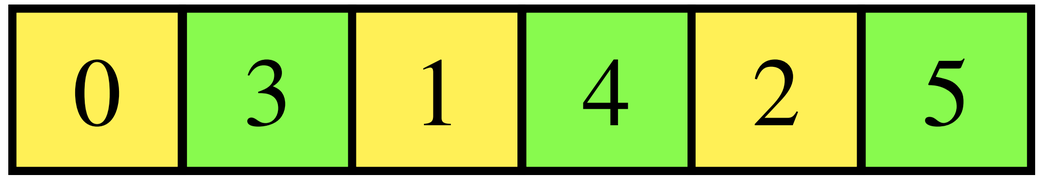


cache miss!

printout of values: 0 1 2 3 4 5

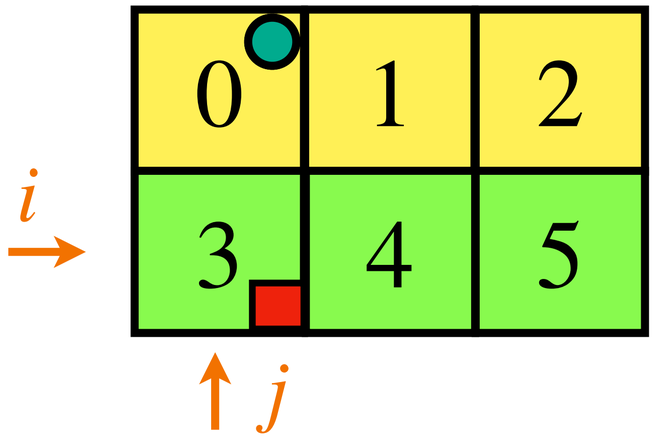


cache misses: 4



Column-major order

Memory address formulas



rows = 2
cols = 3

Row-major order:

0	1	2	3	4	5

index = 3

$\text{index} = i \cdot \text{cols} + j$

$i = 1$ $j = 0$ $\text{index} = 1 \cdot 3 + 0$

Column-major order:

0	3	1	4	2	5

index = 1

$\text{index} = j \cdot \text{rows} + i$

$i = 1$ $j = 0$ $\text{index} = 0 \cdot 2 + 1$

Higher dimensions

Suppose a D-dimensional tensor with shape $N_0 \times N_1 \times \ldots \times N_{D-1}$

dim d is indexed by i_d | $i_d \in \{0, \ldots, N_d - 1\}$ | $d \in \{0, \ldots, D - 1\}$

Row-major order:

$$\text{index} = i_{d-1} + N_{d-1} \cdot (i_{d-2} + N_{d-2} \cdot (\ldots + N_1 i_0))$$

Last dimension is contiguous ("moves fastest")

Column-major order:

$$\text{index} = i_0 + N_0 \cdot (i_1 + N_1 \cdot (\ldots + N_{d-2} i_{d-1}))$$

Zeroth dimension is contiguous

Conventions in different languages

Row-major order:

C
C++
Pascal
ML Libraries
PyTorch
TensorFlow

Column-major order:

FORTRAN
R
MATLAB
Julia

Alternative: Iliffe vectors

Many libraries (e.g. NumPy) support both