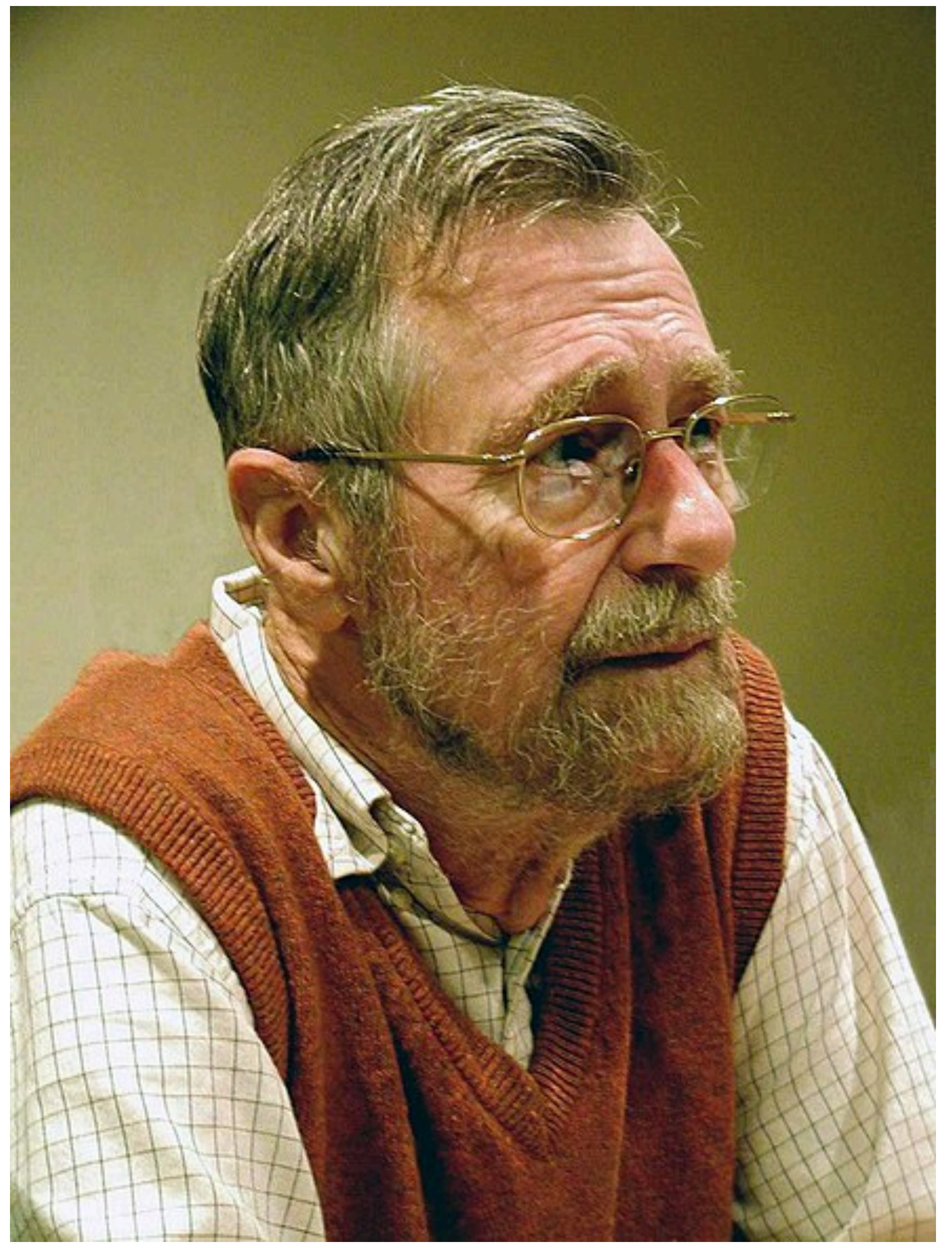
Should numbering **start at 1**?

1, 2, 3, 4, ...

Or should it **start at 0**?

0, 1, 2, 3, 4, ...

"Why numbering should start at zero"
EWD 831(1982)



Edsger W. Dijkstra

Image credit/Reference:
https://commons.wikimedia.org/wiki/File:Edsger_Wybe_Dijkstra.jpg
E. W. Dijkstra, "Why numbering should start at zero" (1982)

(this won't do)

Consider the sequence $2, 3, \ldots, 12$ ← Sequence has 11 elements

A $2 \leq i < 13$ — difference $13 - 2 = \mathbf{11}$ ✓

B $1 < i \leq 12$ — difference $12 - 1 = \mathbf{11}$ ✓

C $2 \leq i \leq 12$ — difference $12 - 2 = \mathbf{10}$ ✗

D $1 < i < 13$ — difference $13 - 1 = \mathbf{12}$ ✗

Are some choices *better than others*?

Sequence length

A **B** superior to **C** **D**

Reference/Image credit:
E. W. Dijkstra, "Why numbering should start at zero" (1982)
https://commons.wikimedia.org/wiki/File:Edsger_Wybe_Dijkstra.jpg

(this won't do)

Consider the sequence $2, 3, \ldots, 12$

A $2 \le i < 13$

$2 \le i < (8)$ $(8) \le i < 13$ ✓

B $1 < i \le 12$

$1 < i \le (7)$ $(7) < i \le 12$ ✓

C $2 \le i \le 12$

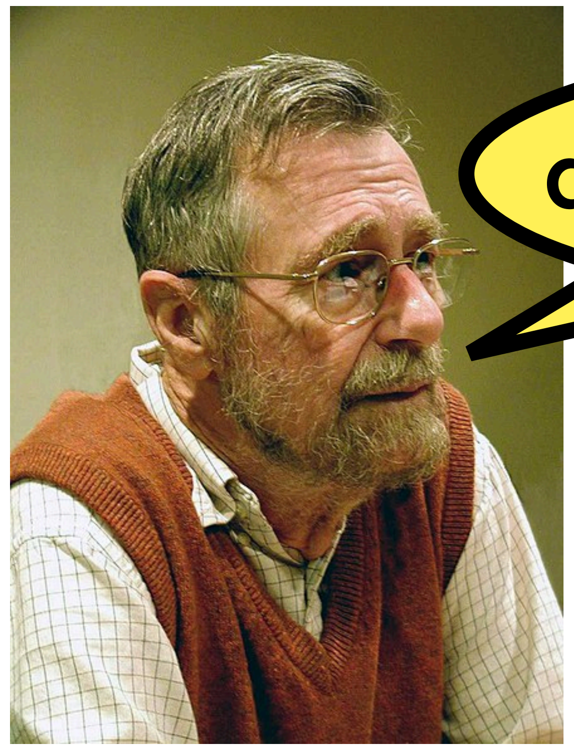
$2 \le i \le (7)$ $(8) \le i \le 12$ ✗

D $1 < i < 13$

$1 < i < (8)$ $(7) < i < 13$ ✗

Are some choices *better than others*?

Adjacency: **A** **B** superior to **C** **D**



another thing too

A vs **B** ?

Reference/Image credit:
E. W. Dijkstra, "Why numbering should start at zero" (1982)
https://commons.wikimedia.org/wiki/File:Edsger_Wybe_Dijkstra.jpg

There is a smallest natural number, s (typically 0 or 1)

or 2 for classical Greeks

Suppose we want to represent $s, s+1, s+2$

A $s \leq i < s+3$

B $s-1 < i \leq s+2$

not a natural number

C $s \leq i \leq s+2$

D $s-1 < i < s+3$

Are some choices *better than others?*

natural lower boundary

A **C** superior to **B** **D**

Image credit/Reference:
E. W. Dijkstra, "Another Filler of the YoP Institute" (1988)
https://commons.wikimedia.org/wiki/File:Edsger_Wybe_Dijkstra.jpg
E. W. Dijkstra, "Why numbering should start at zero" (1982)

Consider the empty sequence obtained by shrinking sequences starting from s

A $s \leq i < s$

B $s-1 < i \leq s-1$

not a natural number

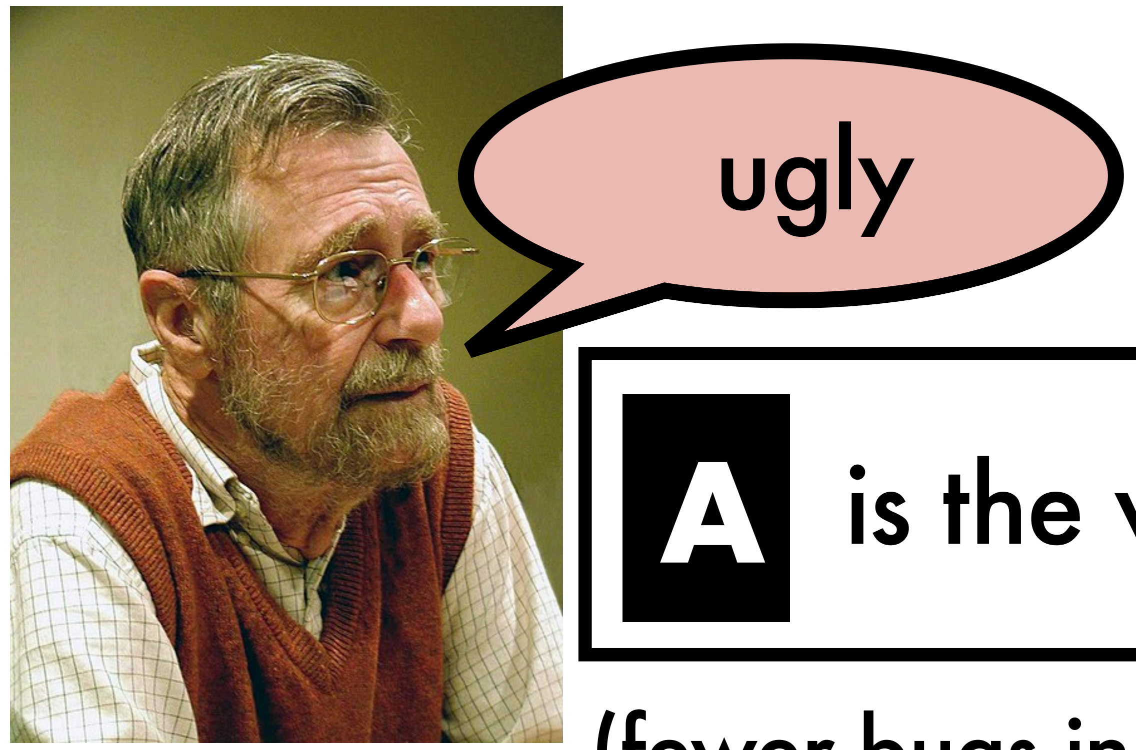
C $s \leq i \leq s-1$

D $s-1 < i < s$

Are some choices *better than others?*

natural upper boundary

A **D** superior to **B** **C**



A is the winner

(fewer bugs in practice)

Image credit/Reference:
https://commons.wikimedia.org/wiki/File:Edsger_Wybe_Dijkstra.jpg
E. W. Dijkstra, "Why numbering should start at zero" (1982)

For a sequence of N elements, which subscript denotes the **first element**?

We've agreed to write: $x \leq i < y$

0	$0 \leq i < N$
1	$1 \leq i < N + 1$

With **0** the value of i equals the number of elements **before it** in the sequence

Moral of story: we should regard zero as a "most natural" number

Conclusion:

numbering should start from **zero**

Image credit/Reference:
https://commons.wikimedia.org/wiki/File:Edsger_Wybe_Dijkstra.jpg
E. W. Dijkstra, "Why numbering should start at zero" (1982)